Noise, Signal Detection

1. Band Levels

1.1. Band Intensity Level

① How to make

Let W be the bandwidth of a desired band intensity level

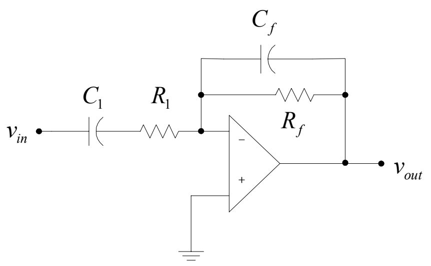
f_L : lower limit of the band

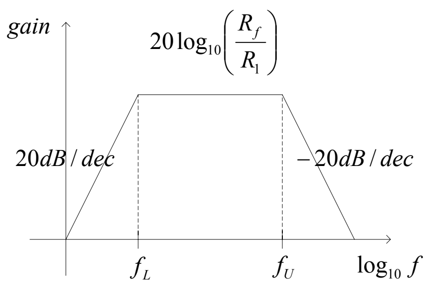
f_U : upper limit of the band

$$\therefore \quad W = f_U - f_L$$

construct a bandpass filter for the band (f_L, f_U)

(with an analog circuit or by digital simulation)

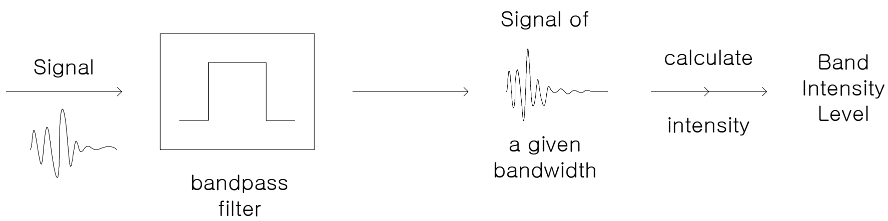




$$f_U = \frac{1}{2\pi C_f R_f} \qquad f_L = \frac{1}{2\pi C_1 R_1}$$

for digital filters consult books on digital signal processing

Idealized model



② Octave band, Linear band

Linear band : constant bandwidth independent of center frequency

$\therefore$ center frequency is determined by

$$f_c = \frac{f_U + f_L}{2}$$ i.e. constant bandwidth in linear scale

Noise, Signal Detection

Octave band : the upper-limit frequency is $2^{\frac{1}{N}}$ times of the lower limit frequency

i.e. Octave band : $f_U = 2 f_L$

$\frac{1}{3}$ octave band : $f_U = 2^{\frac{1}{3}} f_L$

The center frequency is defined $f_c = \sqrt{f_U f_L} = 2^{\frac{1}{2N}} f_L$

Octave band : bandwidth $BW = f_U - f_L = f_c\left(\sqrt{2} - \frac{1}{\sqrt{2}}\right) = \frac{f_c}{\sqrt{2}}$

$\frac{1}{N}$ octave band : bandwidth $BW = f_U - f_L = f_c\left(2^{\frac{1}{2N}} - \frac{1}{2^{\frac{1}{2N}}}\right)$

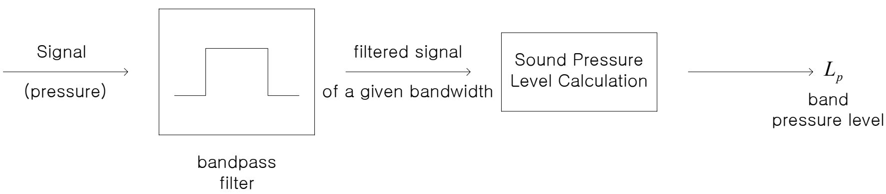
$\frac{1}{3}$ octave band : bandwidth $BW = f_c\left(2^{1/6} - 2^{-1/6}\right)$

Table 1-1 One-third Octave bands

f_L	f_c	f_U	A-weighting(dB)	C-weighting(dB)
18.0	20	22.4	-50.5	-6.2
22.4	25	28.0	-44.7	-4.4
28.0	31.5	35.5	-39.4	-3.0
35.5	40	45	-34.6	-2.0
45	50	56	-30.2	-1.3
56	63	71	-26.2	-0.8
71	80	90	-22.5	-0.5
90	100	112	-19.1	-0.3
112	125	140	-16.1	-0.2
140	160	180	-13.4	-0.1
180	200	224	-10.9	0
224	250	280	-8.66	0

280	315	355	-6.6	0
355	400	450	-4.8	0
450	500	560	-3.2	0
560	630	710	-1.9	0
710	800	900	-0.8	0
900	1000	1120	0	0
1120	1250	1400	+0.6	0
1400	1600	1800	+1.0	-0.1
1800	2000	2240	+1.2	-0.2
2240	2500	2800	+1.3	-0.3
2800	3150	3550	+1.2	-0.5
3550	4000	4500	+1.0	-0.8
4500	5000	5600	+0.5	-1.3
5600	6300	7100	-0.1	-2.0
7100	8000	9000	-1.1	-3.0
9000	10000	11200	-2.5	-4.4
11200	12500	14000	-4.3	-6.2
14000	16000	18000	-6.6	-8.5
18000	20000	22400	-9.3	-11.2

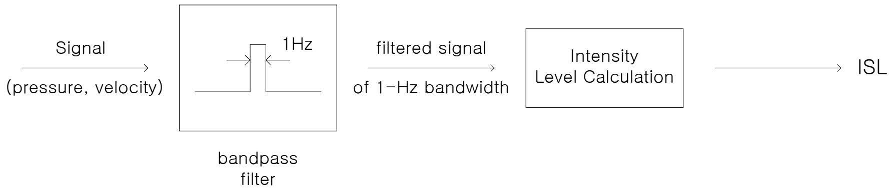
1.2. Band Pressure Level



2. Spectrum Levels

2.1. Intensity Spectrum Level (ISL)

① Band Intensity Level with 1Hz bandwidth



② Band Intensity Level with linear bandwidth

2.2. Intensity Spectrum Level & Other Band Levels

① Linear Band Intensity Level with foxed bandwidth BW

Assume there exist Intensity “Spectral Density” Ι such that

Intensity Band Level

$$L_I(f_L, f_U) = 10\log_{10}\frac{\int_{f_L}^{f_U} \mathrm{I}(f)df}{I_{ref}}$$

Then $ISL = 10\log_{10}\frac{\int_{f_L}^{f_{L+1}} \mathrm{I}(f)df}{I_{ref}}$

Band Intensity Level with bandwidth BW

$$L_{IB} = 10\log_{10}\frac{\int_{f_L}^{f_{L+BW}} \mathrm{I}(f)df}{I_{ref}} = 10\log_{10}\frac{BW\int_{f_L}^{f_{L+1}} \mathrm{I}(f)df}{I_{ref}}$$

$$= 10\log_{10}\frac{\int_{f_L}^{f_{L+1}} \mathrm{I}(f)df}{I_{ref}} + 10\log_{10} BW$$

$$= ISL + 10\log_{10} BW$$ if BW is an positive integer

And the total intensity of the noise $I = \int_0^\infty \mathrm{I}(f)df$

$$IL = 10\log_{10}\frac{\int_0^{\infty} I(f)df}{I_{ref}} = 10\log_{10}\frac{\sum I_i}{I_{ref}} = 10\log_{10}\left(\sum_i 10^{L_{Ii}}\right)$$

② $\frac{1}{N}$ **Octave Band Intensity Level**

$$BW = \left(2^{\frac{1}{N}} - 1\right) f_L$$

$$\therefore \quad L_{IB} = ISL + 10\log_{10}\left\{\left(2^{\frac{1}{N}} - 1\right) f_L\right\}$$

2.3. White Noise

$I = I_0$ (constant)

for Linear-Band Intensity Level

$$L_{IB} = 10\log_{10}\left(I_0 / I_{ref}\right) + 10\log_{10} BW$$

Since BW is constant $L_{IB_i} = L_{IB_j}$ for $\forall\, i, j$

for $\frac{1}{N}$ Octave Band Intensity Level

$$L_{IB_i} = 10\log_{10}\left(I_0 / I_{ref}\right) + 10\log_{10}\left(2^{\frac{1}{N}} - 1\right) f_{L_i}$$

$$L_{IB_{i+1}} = 10\log_{10}\left(I_0 / I_{ref}\right) + 10\log_{10}\left(2^{\frac{1}{N}} - 1\right) f_{L_{i+1}}$$

Since $f_{L_{i+1}} = 2^{\frac{1}{N}} f_{L_i}$

$$\therefore \quad L_{IB_{i+1}} = 10\log_{10}\left(I_0 / I_{ref}\right) + 10\log_{10} 2^{\frac{1}{N}}\left(2^{\frac{1}{N}} - 1\right) f_{L_i}$$

$$\therefore \quad L_{IB_{i+1}} - L_{IB_i} = 10\log 2^{\frac{1}{N}}\left(2^{\frac{1}{N}} - 1\right) f_{L_i} - 10\log\left(2^{\frac{1}{N}} - 1\right) f_{L_i}$$

$$=10\log\frac{2^{\frac{1}{N}}\left(2^{\frac{1}{N}}-1\right)f_{L_i}}{\left(2^{\frac{1}{N}}-1\right)f_{L_i}}=\frac{10\log 2}{N}\approx\frac{3.01}{N}(dB)$$

Therefore $\frac{1}{N}$ Octave Band Intensity Level of white noise increases with center frequency

2.4. Pink Noise

A noise whose octave band intensity levels are constant

i.e. $L_{IB_i}=10\log_{10}\frac{\int_{f_{L_i}}^{2f_{L_i}} I(f)df}{I_{ref}}=ISL_0$ (constant)

$$\therefore\quad \int_{f_{L_i}}^{2f_{L_i}} I(f)df = I_i 2f_{L_i}=2I_i f_{L_i}=const.$$

$$\therefore\quad I_i=\frac{const}{2f_{L_i}}$$

2.5. Pressure Spectral Density, Pressure Spectrum Level

Let $P^2(f)$ be the pressure spectral density of a noise

Pressure spectrum level $10\log_{10}\frac{\int_{f_L}^{f_L+1}P^2(f)df}{P_{ref}{}^2}$

Band pressure level with bandwidth BW

$L_{P_B}=PSL+10\log_{10}BW$ if BW is a positive integer

The total sound pressure level of a noise

$$SPL=L_P=10\log_{10}\frac{\int_0^\infty P^2(f)df}{P_{ref}{}^2}=10\log_{10}\frac{\sum P_i^2}{P_{ref}{}^2}=10\log_{10}\left(\sum 10^{L_{P_i}}\right)$$

2.6. Intensity Spectral Density, Pressure Spectral Density

Auto-correlation of pressure

$$R_p(\tau)=E[p(t)p(t+\tau)]=\lim_{T\to\infty}\left[\frac{1}{T}\int_0^T p(t)p(t+\tau)dt\right]=E\left[\frac{1}{T}\int_0^T p(t)p(t+\tau)dt\right]$$

power spectral density of pressure

$$P^2(f) = \int_{-\infty}^{\infty} R_p(\tau) e^{-j2\pi f\tau} d\tau$$

By the inverse Fourier transfer

$$R_p(\tau) = \int_{-\infty}^{\infty} P^2(f) e^{j2\pi f\tau} d\tau$$

Since $R_p(0) = \frac{1}{T}\int_0^T \{p(t)\}^2 dt = \int_{-\infty}^{\infty} P^2(f) df$: This is the variance of the pressure signal!

Let $\tilde{p}(t)$ be the signal obtained by filtering the pressure signal through a bandpass filter whose Fourier transform is

$$U(f_L, f_U) = \begin{cases} 1 & \text{if } f_L \le f \le f_U \\ 0 & \text{otherwise} \end{cases}$$

the power spectral density of the processed signal

$$\tilde{P}^2(f) = U^2(f_L, f_U) \cdot P^2(f)$$

$$R_p(0) = \int_{-\infty}^{\infty} \tilde{P}^2(f) df = \int_{-\infty}^{\infty} U^2(f_L, f_U) P^2(f) df = \int_{f_L}^{f_U} P^2(f) df$$

Since $P^2(f)$ satisfies all the properties required for pressure spectral density, we may define $P^2(f)$ as pressure spectral density

Intensity spectral density can be defined as cross-spectral density of pressure and particle velocity.

i.e.

$$\mathrm{I}(f) = \int_{-\infty}^{\infty} R_{pu}(\tau) e^{-j2\pi f\tau} d\tau$$

where $R_{pu}(\tau) = E[p(t)u(t+\tau)] = E\left[\frac{1}{T}\int_0^T p(t)u(t+\tau) dt\right]$

$$= \lim_{T\to\infty}\left[\frac{1}{T}\int_0^T p(t)u(t+\tau) dt\right]$$

Note that

$$R_{pu}(0) = \lim_{T\to\infty}\left[\frac{1}{T}\int_0^T p(t)u(t) dt\right]$$: intensity of noise

$$= E\left[\frac{1}{T}\int_0^T p(t)u(t)dt\right] \quad \text{: intensity of noise}$$

$$= E\left[\int_0^\infty \mathrm{I}(f)e^{j2\pi f\tau}df\bigg|_{\tau=0}\right]$$

$$= E\left[\int_0^\infty \mathrm{I}(f)df\right] = \int_0^\infty E[\mathrm{I}(f)]df$$

∴ The cross spectral density of pressure and particle velocity is the intensity spectral density

3. Noise & Noise Level

3.1. What Can Represent Noisiness?

We have to find a physical quantity which can be measurable as well as represent noisiness

We have to study human's hearing characteristics and psychological reaction

Ears should be investigated to find their characteristics as a sensor in addition to their physical structure

3.2. It is well-known that noisiness is related to the intensity of the noise

But it is not directly proportional to the intensity

Frequency contents of a noise can affect noisiness

Feeling of noisiness is dependent on multi-variables

Now we investigate various aspects

4. Detecting Signal in Noise

Detection of a signal in the presence of noise ultimately reduces to a statistical problem

Probability of a true detection $P(D) = \int_{A_T}^{\infty} \rho_{S,N} dA$

Probability of a false alarm $P(FA) = \int_{A_T}^{\infty} \rho_N dA$

ROC (receiving operator characteristic)

A_T : threshold criterion

Ditectability index $d' = \dfrac{A_{S,N} - A_N}{\sigma}$ if σ is the same for $\rho_{S,N}$ & ρ_N

A_N : average of noise $\int_{-\infty}^{\infty} \rho_N A dA$

$A_{S,N}$: average of signal + noise $\int_{-\infty}^{\infty} \rho_{S,N} A dA$

It is important to determine $P(FA)$ & $P(D)$ because the large value of $P(FA)$ diminishes the credibility of the results "detection" by a subject

5. Detection Threshold

Detection Threshold

for a specified $P(FA)$ Define the detection threshold as the ratio of signal power S in some bandwidth W to noise power N which guarantees a given $P(D)$

In decibel measure $DT = 10\log\dfrac{S}{N}$

① **Signal is known**

Detectability index $d' = \sqrt{2w\tau S/N}$

τ : duration of signal

w : bandwidth

$\therefore \quad DT = 10\log_{10}\left[(d')^2/(2w\tau)\right]$

② **Signal is unknown**

Detectability index $d' = \sqrt{w\tau S/N}$

τ : duration of signal

w : bandwidth

$$\therefore \quad DT = 5\log_{10}\left[(d')^2/w\tau\right]$$

6. Ear

6.1. Outer Ear

Pinna (horn)

Auditory canal (2.5cm long, 0.1cm diameter) → 3kHz resonance. 10dB gain (2~6kHz)

Tympanic membrane (eardrum)

6.2. Middle Ear

Three ossicles(bones) : malleus(hammer), incus(anvil), stapes(stirrup)

Eustachian tube

Oval window

6.3. Inner Ear

Vestibule(entrance chamber)

Semicircular canals

Cochlea : upper gallery(scala vestibuli), lower gallery(scala tympani), helicotrema

Round window

Cochlea : bony large, auditory nerve

basilar membrane, tectorial membrane, Reissner's membrane

spiral ligament, cochlea duct, endolymphatic fluid, perilympatic fluid

Organ of corti, hair cells

The peak displacement amplitudes of the basilar membrane shift from stapes to Apex as frequency of a given pure tone signal increases

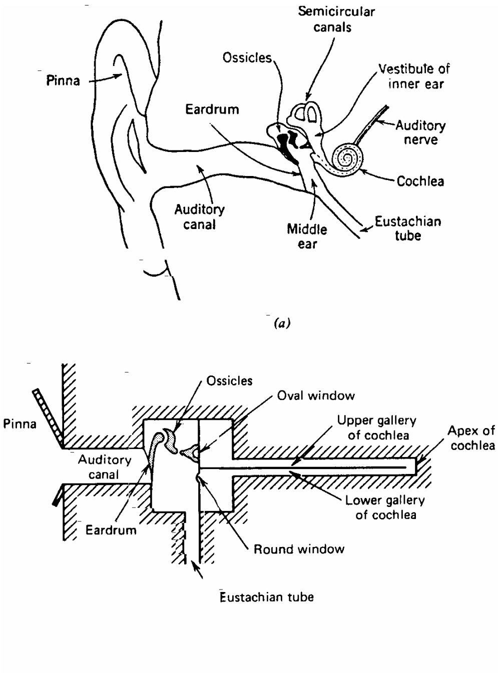


Fig 6.1 Sketch of the ear

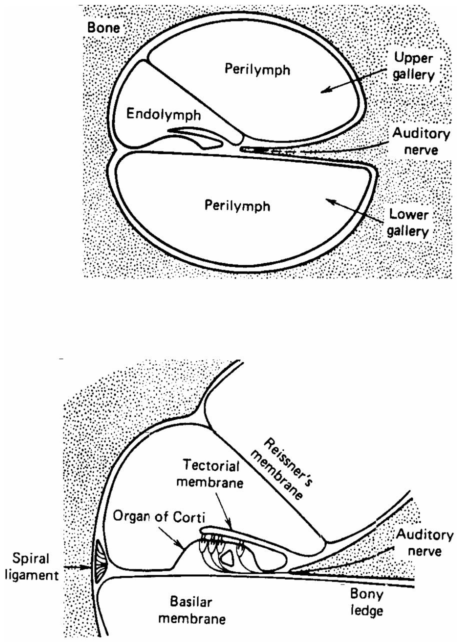


Fig 6.2 Cross section of the cochlea duct

7. Properties of the Human Ear

7.1. Threshold

A tone of duration τ $\begin{cases} 0 \le \tau < 0.3\text{sec} & \text{loudness} \propto \tau \\ \tau > 3\text{sec} & \text{loudness decreases with } \tau \end{cases}$

maximum sensitivity at 4kHz

① **TTS (temporary threshold shift)**

② **PTS (permanent threshold shift)**

③ **Differential thresholds**

Differential threshold for intensity determination

: greatest sensitivity at 3 beats per sec (beating)

40dB above the threshold sensitivity $\begin{cases} \text{less than 2dB for } f > 10\text{kHz} \\ \text{less than 1dB } 100\text{Hz} < f < 1\text{kHz} \end{cases}$

difference limen

: ability to discriminate between two sequential signals of nearly the same frequency

Greatest sensitivity $\begin{cases} 1\text{Hz for } f \le 100\text{Hz} \\ 5\text{Hz for } 100\text{Hz} < f \le 1\text{kHz} \\ 70\text{Hz for } f = 10\text{kHz} \end{cases}$

about 0.7% frequency change can be detect

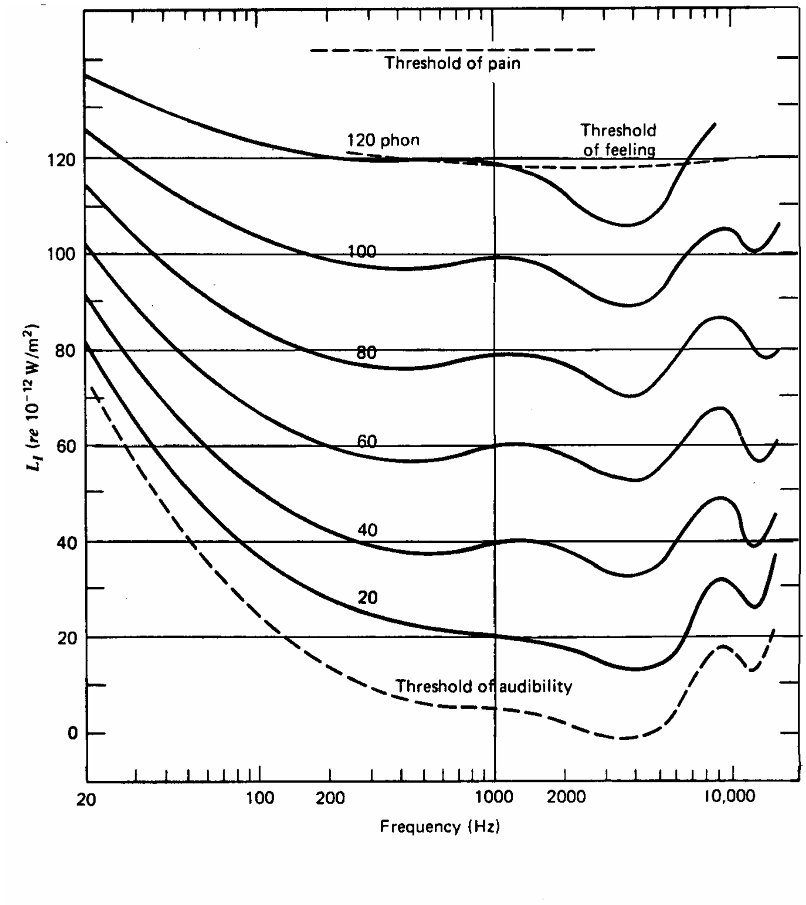


Fig 7.1 Threshold and free-field, equal-loudness-level contours for pure tones with subject facing the source.

7.2. Equal Loudness Level Contours

Loudness level : phon (see the figure 11.9)

7.3. Critical Bandwidth

The masking of a tone by a broadband noise is independent of the noise bandwidth until the bandwidth became smaller than some critical value that depends on the frequency of the tone

Critical ratio : $DT = 0 \quad \Rightarrow \quad w_{cr} = S/N_1$

S : signal power

N_1 : noise power per hertz

Critical bandwidth : determined by the experiments such that the loudness of a band of noise is observed as a function of bandwidth whole the overall noise level is held constant

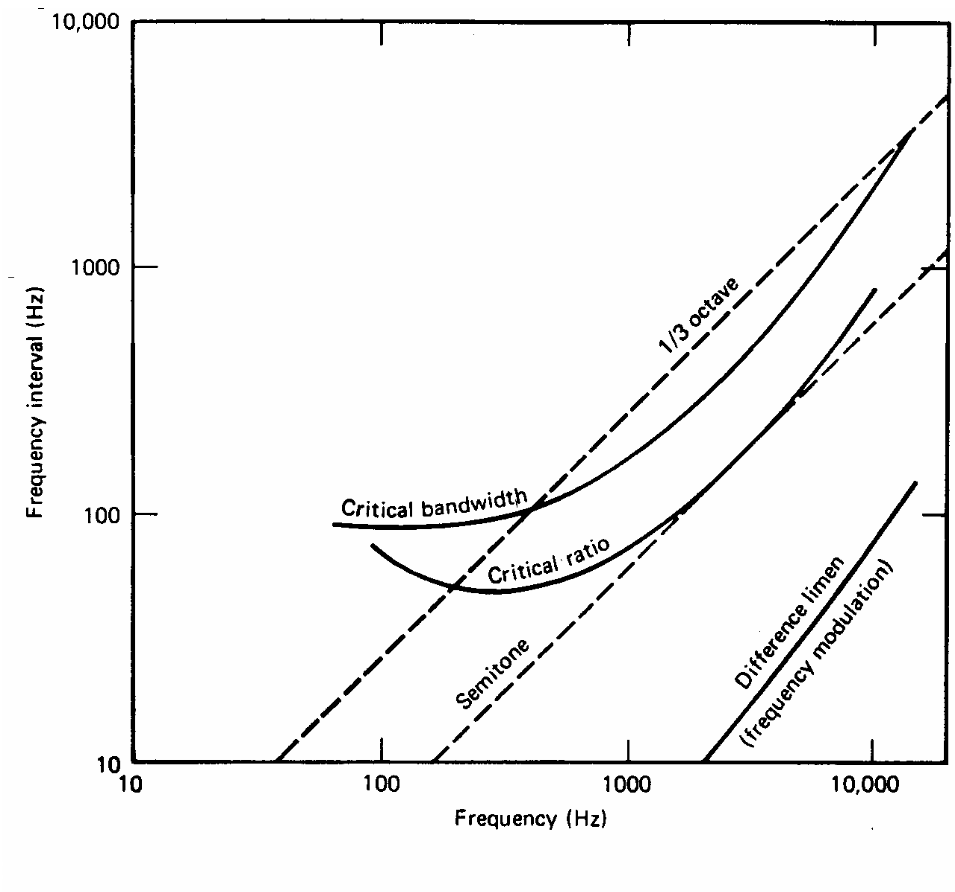


Fig 7.2 Critical bandwidths of the ear

7.4. Masking

The frequency range over which there is appreciable masking increases with the L_I of the masker, the increase being greater for frequencies above that of the masker.

Masking of pure tones by a band of noise narrower than w_{cb} is essentially the same as that of an equally intense pure tone having the same frequency as that at the center of the band

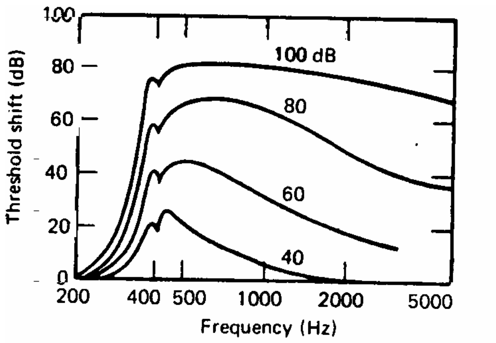


Fig 7.3 Masking by a pure tone at 400Hz

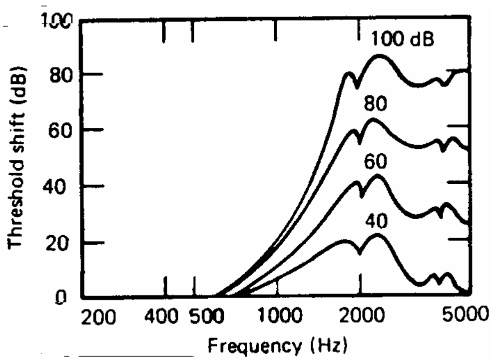


Fig 7.4 Masking by a pure tone at 2kHz

7.5. Cochlea Nonlinear Effects

Two tones of frequencies f_1 & f_2

① f_1 is close enough to f_2 of $f_1 \cong f_2$

ear perceive a tone at $f = \dfrac{f_1 + f_2}{2}$ with beating frequency $|f_2 - f_1|$

(until less than 5 beats per sec, i.e. $(f_2 - f_1) < 5\text{Hz}$)

② $5\text{Hz} < f_2 - f_1 < 10\text{Hz}$: throbbing

③ $15\text{Hz} < f_2 - f_1 < 30\text{Hz}$: roughness

④ $f_2 - f_1 >$ critical bandwidth : independent

⑤ Combination tones : $f(\text{combonation}) = |mf_2 \pm nf_1| \quad m, n = 1, 2, 3, \cdots$

Generation of combination tones is evidence of nonlinear effects on ears

Nonlinear property can be identified by observing aural harmonics

for a tone $f = 100\text{Hz}$, loudness $= 20\text{phon}$ second harmonics are sensed by the subject

7.6. Nonlinear Processing Effects

for two tones $f_1 / f_2 \cong \dfrac{1}{2}$, $\Rightarrow$ beating is sensed by human ears

<Although two tones are presented separately to each ear, beating is still sensed>

Hence, it cones from nonlinear processing effects, in the processing of information in the relevant centers of the brain

8. Pitch and Frequency

$100\text{Hz} \le f \le 300\text{Hz}$ L_N increases $\Rightarrow$ pitch decreases

$500\text{Hz} \le f \le 3000\text{Hz}$ pitch is independent of L_N relatively

$f \ge 4000\text{Hz}$ L_N increases $\Rightarrow$ pitch increases

mel : unit of pitch

reference frequency : 1kHz, 60phon : 1000mel

see the figure 1.14 (pp 273)

9. Loudness Level and Loudness

Loudness Level L_N in phon

Loudness N in sone

In reality N is not proportional to L_N in $L_N < 40\text{phon}$

For $L_N \ge 40\text{phon}$ $N \cong 0.046 \times 10^{\frac{L_N}{30}}$

Since $L_I = 10\log\left(I/10^{-12}\right)$, at 1kHz ($L_I = L_N$ at 1kHz)

$$N(1\text{kHz}) = 460\sqrt[3]{I}$$

Using this formula, we consider the following three cases.

① tones lying within one critical bandwidth

$$N(\text{critical band}) = 460F(f)\left(\sum_i I_i\right)^{\frac{1}{3}}$$

② tones differing by more than the relevant critical bands

$$N = \sum_i N_i$$

③ tones are considerably different in their loudness, or widely separated on frequency, evaluation of loudness becomes difficult, often tending to be base on the loudest of tones

34 sone = 84 phon

If we use formula in ①

$$L_I = 68\text{dB}$$

Table 9-1 Sample calculation of loudness

Frequency f (Hz)	Intensity Level L_I (dB)	Loudness Level L_N (phon)	Loudness N (sone)
125	60	55	3.2
250	60	62	5.4
500	60	63	5.9
1000	60	60	4.7
2000	60	62	5.4
4000	60	69	9.3
Total			33.9